# Three-Layer-Isotropic Skull Conductivity Representation in the EEG Forward Problem using Spherical Head Models

E. Cuartas-Morales[1] Hans Hallez[2] Bart Vanrumste[3] G. Castellanos-Dominguez[1]

***Abstract*— We investigate influence of different conductivity models within a framework of electroencephalogram (EEG) source localization on white matter and skull areas. Particularly, we investigate five different spherical models having either isotropic or anisotropic conductivity for both considered areas. To this end, the anisotropic finite difference reciprocity method is used in solving the EEG forward problem. We evaluate a numeric skull conductivity modeling, in terms of the minimum dipole localization/orientation error. As a result, two skull models reach the lowest dipole localization error (less than $6\,mm$), namely, single anisotropic layer and three isotropic layers (hard bone/spongy bone/hard bone). Additionally, two different electrode configurations ($10-20$ and $10-10$ electrodes) are tested showing that the error decreases almost twice for the latter one, although computational burden significantly increases.**

## I. Introduction

Nowadays, several methodologies have been proposed to analyze brain structures with high precision for efficient surgery planning (mostly, in Epilepsy or Parkinson diseases) or to perform general brain studies. These computational-based methodologies are mainly supported on noninvasive measurements (e.g., electroencephalogram - EEG, magnetic resonance imaging - MRI, or computed tomography) and are used for diagnosis and preoperative brain surgery stages being, in most cases, the only suitable analysis tools due to the high risk of alternative invasive interventions [9].

Meanwhile, noninvasive methods are commonly focused on location of neural activity sources inducing electrical potentials in the head. Those potentials can be measured by electrodes placed directly on the scalp (i.e., EEG). In this regard, the source localization EEG problem is divided into the following two subsequent tasks: *i*) The forward problem calculating electrode potentials on the scalp for a provided source configuration [6]. *ii*) The inverse problem estimating source parameters from electrode potentials [5]. The latter problem solution usually results in an iterative task. The solution is reached assuming that the electrode potentials measured on the scalp are similar to those calculated by a reference model.

This work is carried out under grants: *Programa Nacional de Formacion de Investigadores Generacion del Bicentenario, 2012, conv:528*, and the research project `111045426008`; both funded by COLCIENCIAS

[1] Universidad Nacional de Colombia, Campus La Nubia. Signal Processing and Recognition Group, `ecuartasmo@unal.edu.co`, `ecuartasmo@unal.edu.co`

[2] Department of Electronic Design and Realisation, Research Group Reliability in Mechatronics in ICT (ReMI), KU Leuven-KULAB, Campus Oostende, Zeedijk 101, 8400 Oostende, Belgium, `hans.hallez@kuleuven.be`

[3] AdvISe, KU Leuven campus Geel, ESAT-STADIUS, KU Leuven, iMinds Medical Information Technology department, Belgium, `Bart.Vanrumste@kuleuven.be`

On the other hand, both the skull and white matter have strong anisotropic conductivity highly affecting performance of source localization [7]. From the clinical point of view, skull composition is assumed to have three embedded stratums (a spongy layer between another two hard layers), each one having different conductivity. Thus, anisotropic skull conductivity can be described by a three-layers-isotropic conducting model, namely, two separate compact zones plus a soft one [1]. However, accuracy of this model has been tested against numerical approximated methods (like finite difference method [6] or finite element method [10]) that may jeopardize the error calculation of reconstructed head source parameters [2].

To cope with this issue, we develop a 3-layer-isotropic spherical model that is further compared against the baseline analytical representation suggested in [3], when conductivity of white matter and skull is modeled as anisotropic. Particularly, five different skull conductivity modelings are compared in terms of the minimum dipole localization/orientation error, using the *anisotropic finite difference reciprocity method* (AFDRM) to calculate the numeric potentials against the analytical solution described in [3]. We use three different skull conductivity models, isotropic, anisotropic, and the suggested 3-layer isotropic, with anisotropic/isotropic white matter in order to analyse the influence of deep sources, and the different skull models. To consider influence of the used number of electrodes, we also carry out testing of both the baseline anisotropic analytical and 3-layer isotropic skull models using the *10-10* and *10-20* EEG systems.

## II. Methods

### A. Forward Problem

Related to EEG source location, the forward problem estimates the electrode potential field, $\boldsymbol{V}$, placed at a specific point, $(x,y,z)$, on the scalp that is generated due to current sources inside the brain. Sources are modeled as current dipoles described at position $\boldsymbol{r}\in\mathbb{R}^3$ with orientation $\boldsymbol{d}\in\mathbb{R}^3$. The scalar-valued potential $V(x,y,z)\subset\boldsymbol{V}$ on the surface of a conductive volume $x,y,z$ is defined by the Poisson equation as follows:

$$\nabla\left(\mathit{\Sigma}(x,y,z)\nabla V(x,y,z)\right)=I\delta(\boldsymbol{r}-r_1)-I\delta(\boldsymbol{r}-\boldsymbol{r}_2) \quad (1)$$

where $I\in\mathbb{R}$ represents the current dipole magnitude, $\boldsymbol{\Sigma}\in\mathbb{R}^{3\times 3}$ is the conductivity tensor, and $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the two concrete coordinates determining the dipole direction. Notation $\delta(\cdot)$ stands for the delta function.

In case of isotropic volumes, the conductivity $\Sigma(x, y, z)$ is scalar-valued, while in anisotropic case, it becomes a tensor taking the following form:

$$\boldsymbol{\Sigma}_h^{(j)} = \boldsymbol{T}^{(j)\top} \boldsymbol{\Sigma}_s^{(j)} \boldsymbol{T}^{(j)} \tag{2}$$

where $\boldsymbol{\Sigma}_h^j$ is the conductivity head matrix defined in the uniform cartesian coordinate system at the element $j$; $\boldsymbol{T} \in \mathbb{R}^{3\times 3}$ is the orthogonal matrix of unit length eigenvectors that is a rotation transfer matrix from the local to the global coordinate system; $\boldsymbol{\Sigma}_s^{(j)} = \operatorname{diag}(\sigma_{rad}^{(j)}, \sigma_{\tan}^{(j)}, \sigma_{\tan}^{(j)})$ is a diagonal matrix holding the local conductivity values in the tangential, $\sigma_{\tan}^{(j)}$, and radial directions, $\sigma_{rad}^{(j)}$, respectively.

It must be noted that for modelling the anisotropic conductivity of the skull and white matter, we calculate normal vectors to the sphere reconstruction at every spatial point representing the values of the local tangential, $\sigma_{tan}^{(j)}$, and radial conductivity, $\sigma_{rad}^{(j)}$.

Additionally, for modeling anisotropic white matter conductivity, we also use the volume constrain [10]:

$$\sigma_{iso}^3 = \sigma_{rad}(\sigma_{\tan})^2 \tag{3}$$

where $\sigma_{iso}$ is the isotropic conductivity value of the white matter.

### B. Forward Solution

For the numeric case, Eq. (1) is solved using the anisotropic finite difference methodology in a 18-neighborhood representation, as proposed in [6]:

$$\sum_{i=1}^{18} a_i \boldsymbol{\phi}_i - \left( \sum_{i=1}^{18} a_i \right) \boldsymbol{\phi}_0 = I\delta(\boldsymbol{r} - \boldsymbol{r}_1) - I\delta(\boldsymbol{r} - \boldsymbol{r}_2) \tag{4}$$

where the $a_i \in \mathbb{R}$ coefficients holds the conductivity values and ensure the Dirichlet and Neumann boundary conditions [8], $\boldsymbol{\phi}_i \in \mathbb{R}^{1\times N_Z}$ is each discrete potential, being $N_Z$ the non zero voxels where head tissues are present, $\boldsymbol{\phi}_0$ is the potential in the neighborhood origin.

Generally speaking, Eq. (4) results in a linear system $\boldsymbol{A\phi} = \boldsymbol{I}$ with unknown terms, $\boldsymbol{\phi}$, that is solved using *successive over relaxation*. However, its implementation requires high computational burden. Therefore, precalculated reciprocity potentials are employed to speed up the computation of the inverse solution.

### C. EEG dipole source estimation

Within the inverse problem framework, we estimate the pairwise dipole parameters $(\boldsymbol{r}, \boldsymbol{d})$ by calculating the best electrode potentials, in terms of the lowest relative residual energy, $e \in \mathbb{R}^+$, that we minimize as follows [6]:

$$e = \frac{\|\boldsymbol{v}_e(\boldsymbol{r}, \boldsymbol{d}) - \boldsymbol{v}_m(\boldsymbol{r}, \boldsymbol{d})\|_2^2}{\|\boldsymbol{v}_e(\boldsymbol{r}, \boldsymbol{d})\|_2^2} + c(\boldsymbol{r}) \tag{5}$$

where the values $\boldsymbol{v}_e \in \mathbb{R}^{N_d\times 1}$ are the vector of electrode potentials of the analytical reference model; $\boldsymbol{v}_m \in \mathbb{R}^{N_d\times 1}$ are the electrode potential vector estimated by the numerical test models, being $N_d$ the number of considered dipoles; and the term $c(\boldsymbol{r}) \in \mathbb{R}^+$ is a penalization parameter that is set to zero for dipole positions inside the gray matter, otherwise they are very large. Notation $\|\cdot\|_2$ stands for the Euclidean norm.

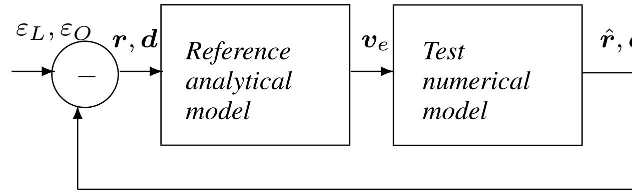


Fig. 1. Diagram of the employed EEG dipole source estimation.

As seen in Fig. 1 that shows the procedure including both the *reference* and *test* models to estimate the dipole error, we initially compute the electrode potentials $\boldsymbol{v}_e$ and then the dipole parameters, $(\hat{\boldsymbol{r}}, \hat{\boldsymbol{d}})$. Namely, we introduce the following two error measures:

– the dipole localization error (DLE),

$$\varepsilon_L = \|\hat{\boldsymbol{r}} - \boldsymbol{r}\|_2$$

– the dipole orientation error (DOE),

$$\varepsilon_O = \arccos\left( \frac{\hat{\boldsymbol{d}}^\top \boldsymbol{d}}{\|\boldsymbol{d}\|_2 \, \|\hat{\boldsymbol{d}}\|_2} \right)$$

## III. Experimental Set-Up

We test the proposed 3-layer-model of skull conductivity within the inverse problem framework that is above explained, including also the white matter conductivity model.The 3-layer-model model is compared against both, the isotropic and anisotropic, conductivity models of Skull and white matter tissues. Therefore, each tested head model holds, at least, five different tissue layers (scalp, skull, gray matter, white matter, and thalamic inner sphere), as shown in the Fig. 2 displaying the concrete spherical disposition used in this work.

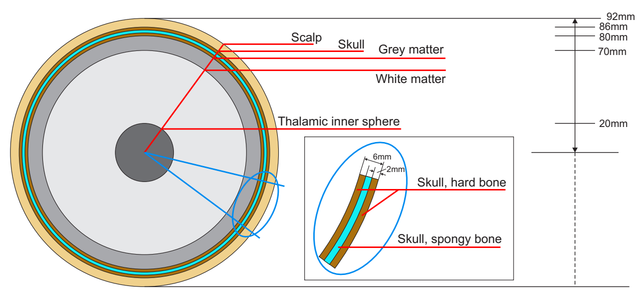


Fig. 2. Spherical head model and layer configuration used during testing.

Therefore, we compare the five spherical head models shown in Table I where the proposed 3-layer-model of skull conductivity are marked in bold. All tested models are generated using the numerical approximation AFDRM method assuming the following set-up values: a 1-*mm*-voxel size resulting in a $186 \times 186 \times 186$ data set, the anisotropic ratio in the skull is fixed as $1 : 1.82$ ((radial: tangential), as used in [7]). Besides, we assume during solution the volume constraint, as defined in [10].

TABLE I

HEAD MODELS INCLUDING CONDUCTIVITY REPRESENTATION OF THE WHITE MATTER AND SKULL

| *Model* | *White Matter* | *Skull* |
|---|---|---|
| A [6] | Isotropic | Isotropic |
| B [6] | | Anisotropic |
| **C** | | **3-layer-Isotropic** |
| D [6] | Anisotropic | Anisotropic |
| **E** | | **3-layer-Isotropic** |

Table II shows all considered values of tissue conductivity as well as the assumed anisotropic ratio (radial:tangential), as suggested in [7]. Testing is carried out using the EEG $10-20$ system (i.e., 19 electrodes and 18 leadpairs) in the above explained reciprocity approach providing 3262312 non-zero potentials and $1\,mm$ voxel size. For every single leadpair calculation, the AFDRM algorithm lasts about 40 minutes using the Intel core i7 processor with 8Gb RAM (not mentioning that the solution must be calculated for every leadpair). In turn, to implement the inverse solution, we assume a set of 6000 dipole sources where the distance between the test dipoles is $5\,mm$. Testing is carried out in three different dipole orientations ($x$, $y$, and $z$), resulting in 18000 calculations.

To get better idea about feasibility of the proposed approach, we employ the EEG *10-10* system with 30 electrodes and 29 lead pairs, but just for the *C* and *D* models, as the most complex ones.

TABLE II

USED TISSUE VALUES FOR CONDUCTIVITY AND ANISOTROPIC RATIO. RATIO VALUE 1:1 IMPLIES ISOTROPIC TISSUE

| *Tissue* | *Conductivity* [*S/m*] | *Anisotropic Ratio* |
|---|---|---|
| Scalp | 0.33 | 1:1 |
| Skull (one-layer) | 0.02 | 1:1.82 |
| Hard bone | 0.0064 | 1:1 |
| Spongy bone | 0.02865 | 1:1 |
| Grey matter | 0.33 | 1:1 |
| White matter | 0.14 | 9:1 |
| Thalamic area | 0.33 | 1:1 |

## IV. RESULTS AND DISCUSSION

Performed values of dipole localization error (*DLE*) are shown in Fig. 3, where each row represents every considered simulation model, while the columns stand for the dipole orientation. The views are the axial cuts of the spherical models and the dots draw the 19 electrodes projected on each actual cut. Table III summarizes the computed mean and standard deviation of the *DLE* and *DOE* values estimated for the gray matter (GM) and the thalamic inner sphere (TL) (models including the proposed 3-layer-isotropic representation are marked in bold). As a result, both models *D* and *E* reach the smallest values of *DLE* and *DOE*. Namely, the *E* model has a maximum *DLE* of $5.74\,mm$ in the gray matter and a maximum *DOE* of $19.14\,deg$ in the thalamic inner sphere. Also, the *model D* has a maximum *DLE* of $7.97\,mm$ in the gray matter and a maximum *DOE* of $18.62\,deg$ in the gray matter, as shown in Fig. 4.

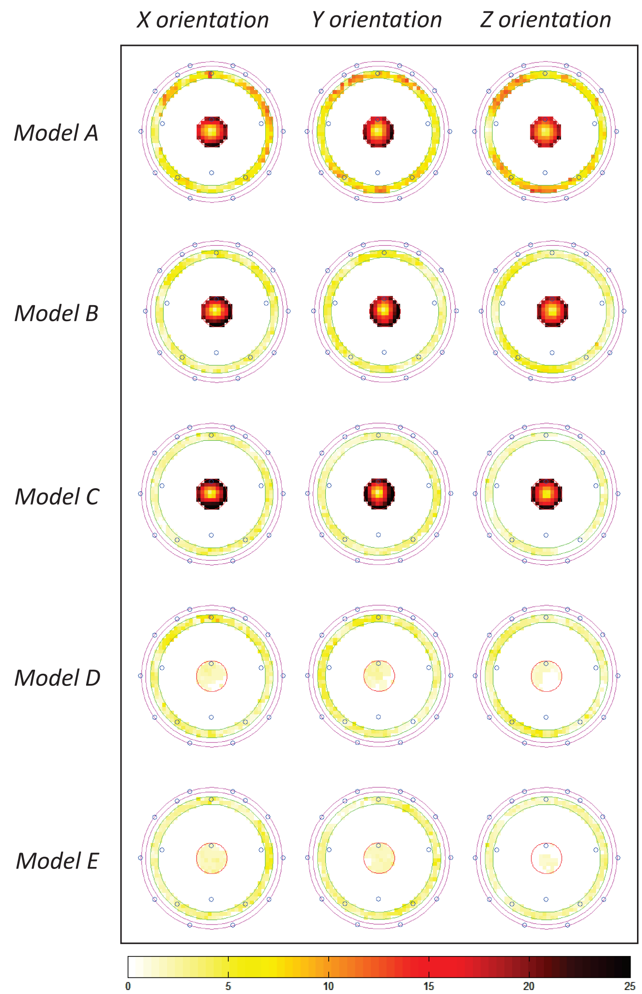


Fig. 3. Spatially distributed *DLE* values for all considered simulation models, estimated for *10-20* EEG system and 19 electrodes.

TABLE III

ESTIMATED LOCALIZATION AND ORIENTATION ERROR VALUES, $\varepsilon_L, \varepsilon_O$, IN THE GRAY MATTER AND THE THALAMIC INNER SPHERE.

| *model* | $GM$ [*mm*] | $GM$ [*deg*] | $TL$ [*mm*] | $TL$ [*deg*] |
|---|---|---|---|---|
| A | $6.52 \pm 2.83$ | $9.18 \pm 7.95$ | $14.85 \pm 5.03$ | $2.76 \pm 2.97$ |
| B | $3.36 \pm 1.63$ | $5.24 \pm 5.98$ | $16.66 \pm 5.60$ | $2.81 \pm 2.91$ |
| **C** | $3.17 \pm 1.72$ | $3.94 \pm 3.96$ | $18.09 \pm 6.06$ | $2.84 \pm 3.05$ |
| D | $2.96 \pm 1.58$ | $3.80 \pm 4.01$ | $1.34 \pm 0.82$ | $1.47 \pm 0.79$ |
| **E** | $2.41 \pm 1.20$ | $3.42 \pm 3.44$ | $1.37 \pm 0.62$ | $1.70 \pm 1.73$ |

Therefore, based on the obtained results shown in Fig. 4 and Table III, we select the models *E* and *D* as having the best skull conductivity representation for further testing. Particularly, we test both models on the EEG $10-10$ system with 30 electrodes. Table IV shows that the *D* model reaches significant diminution of *DLE* value, while *DOE* value gets a bit better for both *D* and *E* models. As seen in Fig. IV and Fig. 3 showing performed *DLE* values for

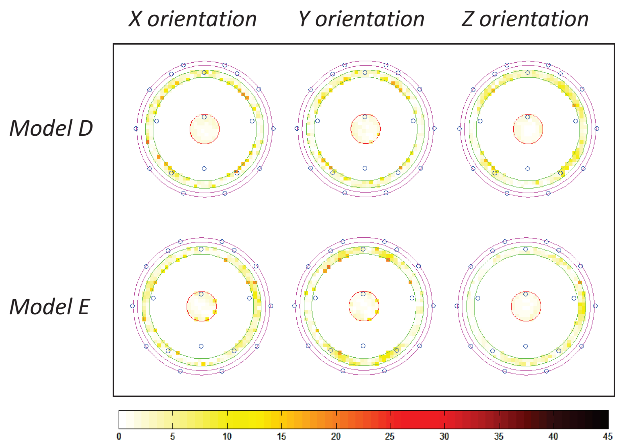


Fig. 4. Performed values of *DOE* computed for the *10-20* EEG system and 19 electrodes.

30 and 19 electrodes, respectively, we can infer that adding more electrodes allows reducing localization error.

| model | $GM$ [mm] | $GM$ [deg] | $TL$ [mm] | $TL$ [deg] |
|---|---|---|---|---|
| D | $1.79 \pm 1.16$ | $3.12 \pm 3.37$ | $0.88 \pm 0.70$ | $1.58 \pm 1.46$ |
| E | $2.39 \pm 1.27$ | $3.06 \pm 3.71$ | $0.85 \pm 0.84$ | $1.45 \pm 0.68$ |

TABLE IV
SUMMARIZED VALUES OF *DLE* AND *DOE* FOR *10-10* EEG SYSTEM AND 30 ELECTRODES

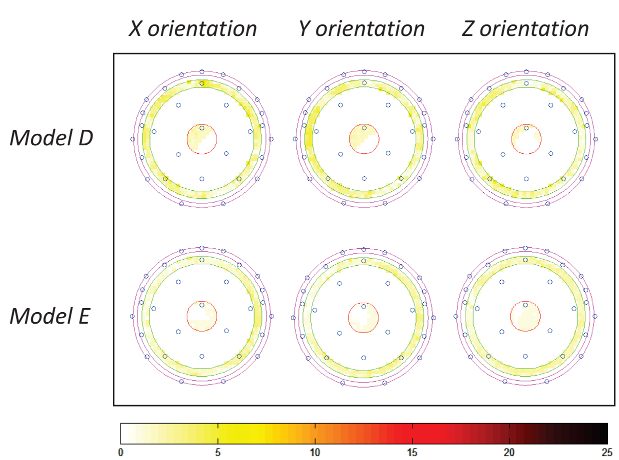


Fig. 5. Performed *DLE* values computed for the *10-10* EEG system and 30 electrodes.

## V. CONCLUSIONS

To deal with anisotropic Skull Conductivity, we develop a 3-layer Isotropic tissue representation within the EEG Forward Problem framework using Spherical Head Models. We carry out comparison, in terms of the dipole localization and orientation errors, with another baseline Skull Conductivity models using the numerical approximation AFDRM method. Particularly, we propose two head models assuming either isotropic (model *C*) or anisotropic (model *E*) conductivity of the white matter. Obtained results on simulated EEG data show that deep sources placed in the thalamic inner sphere have very large DLE and DOE errors (as much as $26\,mm$) using the former model pointing out that the white matter anisotropy should be strongly considered.

In contrast, the latter model turns to be a suitable conductivity representation performing the lowest error values that are close to the baseline E model. However, the proposed threes-layer-isotropic model requires an additional image segmentation step for realistic, patient dependent head models that is far from being an easy task [7].

Another finding through this work is that adding more electrodes (and lead pairs calculations) allows considerably reducing the dipole localization/orientation error, but it implies calculation of more lead-pairs, which increases computational burden of the precalculated potentials in the reciprocal approach.

As a future research, we plan to analyze the EEG source localization errors in realistic head models using state of the art inverse solution such as *multiple sparse priors* approach [4] employing the skull conductivity models of this work. We also want to analyze different anisotropic ratios for the skull and the white matter in order to find the best possible head model.

## REFERENCES

[1] R. Bashar *et al.*, "Effects of White Matter on EEG of Multi-layered Spherical Head Models," in *5th International Conference on Electrical and Computer Engineering*, vol. 00, no. December, 2008, pp. 59–64.
[2] M. Dannhauer *et al.*, "Modeling of the human skull in EEG source analysis." *Human brain mapping*, vol. 32, no. 9, pp. 1383–99, Sep. 2011.
[3] J. C. de Munck and M. J. Peters, "A fast method to compute the potential in the multiphere model," *IEEE transactions on bio-medical engineering*, vol. 40, no. 11, 1993.
[4] K. Friston *et al.*, "Multiple sparse priors for the m/eeg inverse problem," *NeuroImage*, vol. 39, no. 3, pp. 1104 – 1120, 2008.
[5] R. Grech *et al.*, "Review on solving the inverse problem in EEG source analysis." *Journal of neuroengineering and rehabilitation*, vol. 5, p. 25, Jan. 2008.
[6] H. Hallez *et al.*, "Review on solving the forward problem in EEG source analysis," *Journal of NeuroEngineering and Rehabilitation*, vol. 4, 2007.
[7] V. Montes and H. Hallez, "Influence of Skull Inhomogeneities on EEG Source Localization," *Noninvasive Functional Source Imaging of the Brain and Heart & 2011 8th International Conference on Bioelectromagnetism (NFSI & ICBEM), 2011 8th International Symposium on*, pp. 72–76, 2011.
[8] H. I. Saleheen and K. T. Ng, "New Finite Difference Formulations for General Inhomogeneous Anisotropic Bioelectric Problems," *IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING*, vol. 44, no. 9, pp. 800–809, 1997.
[9] N. Voges *et al.*, "Modeling of the Neurovascular Coupling in Epileptic Discharges," *Brain Topography*, vol. 25, no. 2, pp. 136–156, 2011.
[10] C. H. Wolters *et al.*, "Influence of tissue conductivity anisotropy on EEG / MEG field and return current computation in a realistic head model : A simulation and visualization study using high-resolution finite element modeling," *NeuroImage*, vol. 30, pp. 813 – 826, 2006.